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Revista de Cercetare si Interventie sociala

ISSN: 1583-3410 (print), ISSN: 1584-5397 (electronic)

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Revista de cercetare și intervenție socială, 2018, vol. 62, pp. 79-113

The online version of this article can be found at:
www.rcis.ro, *www.doaj.org* and *www.scopus.com*

Published by:
Expert Projects Publishing House

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publishing

On behalf of:
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Department of Sociology and Social Work
and
HoltIS Association

REVISTA DE CERCETARE SI INTERVENTIE SOCIALA
is indexed by Clarivate Analytics (Web of Science) -
Social Sciences Citation Index
(Sociology and Social Work Domains)

The Economic Consequences of Working While Receiving a Full Pension

Quansheng GAO[1], Kang ZHOU[2], Junyong LI[3]

Abstract

This paper investigates the economic consequences of working while receiving a full pension (WRFP). We find that WRFP has crowding out effect on savings of working period and crowding in effect on savings of WRFP period. We show that a unique non-trivial steady-state per capita capital stock of the dynamic system exists and increasing the length of WRFP period and social security contribution rate would increase the speed of capital accumulation reaching its optimal state. The effect of WRFP on welfare gains in the long run is ambiguous and is determined by the elasticity of capital in the two-period overlapping generations (OLG) model, whereas it depends not only on the elasticity of capital but also on the length of WRFP in the three-period OLG model. On the whole, although WRFP has an incentive effect on household agents, welfare losses arisen from its negative externalities exceed welfare gains.

Keywords: working while receiving a full pension, economic effects, welfare implications, OLG model.

Introduction

In many developing countries, working while receiving a full pension (WRFP) is an interesting retirement scenario which is seldom investigated by researchers. WRFP means that household agents can work after retirement without affecting their pension benefits. WRFP has several features different from other paid work after retirement. First, household agents may supply labor for a fraction of their time endowment after the legal retirement age. This characteristic is similar to so-called "bridge employment," a job between a full-time position and permanent withdrawal from the workforce or similar to "un-retirement" that implies a process of exit from the former job to a period of full retirement (Kim, & Daniel, 2000; Maestas, 2010). Next, household agents receive full pensions with few restrictions.

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Finally, household agents' pension benefits will not be affected by their earnings when they work after retirement, which implies they have extra income to save in the WRFP period.

It is worth emphasizing that WRFP investigated in this paper is not a new pension arrangement and is also not a pension transition strategy described by (Cherkashina, 2011). WRFP is an informal pervasive retirement phenomenon accompanied with the developing pension system of many countries. WRFP is distinguished from a flexible retirement plan that combines wage income with a partial state pension and/or occupational pension during a period of gradual retirement. The traditional retirement scenario is characterized by a structural break in the late life cycle from full-time employment to full retirement (Tunga, & Arthur, 2008). Institutions often hamper paid work after the standard retirement age. Only if retirees meet some criteria can they collect the full or even a part of the pension. Different countries regulate work beyond the standard retirement age and the receipt of pension benefits differently (Gruber, & David, 2010). For example, in Spain and Portugal, retirees are not allowed to engage in paid work while receiving a public pension but instead have to completely withdraw from work after retirement. In other countries, some restrictions are imposed on the working while receiving a pension after full retirement. Thus household agents' payments may be affected depending on when they begin receiving pension. In the US, household agents may take up state pension benefits and work simultaneity, but above a threshold earnings amount of about $37,000, pension benefits are withheld in part and paid out later after the pension age (Daniel, 2011).

Those who get full pension while working after retirement age come from three potential sources of retired urban workers and staff, non-employed urban residents and rural residents. For example, in China, according to Chinese pension programs, urban workers and staff don't have to pay much to enjoy generous benefits. Although their replacement rate of pension is over 90%, they continue to work because of their valuable skills and experience (Wang, Wang, & Han, 2013). The pension scheme for Chinese rural residents begins in late 2009， targeting full coverage of the rural residents by 2020 with pension benefits equal to 15%t of rural earnings. Five annual voluntary contribution levels from 100yuan to 500yuan are provided for rural residents. When participants reach retirement age, they can receive a government subsidy of 55yuan/month. The basic pension benefit of social endowment insurance for urban-rural residents has increased to 105yuan/month in 2016. In addition, rural participants can receive monthly pension benefits from personal account with the total amount of funds upon retirement divided by 139. It is obvious that pension benefits of rural residents are far too tiny to support the basic living of the elderly, which has left most of them rely on family support or left them to their own devices. The replacement rate of low-income, middle-income and high-income rural residents is only equal to 24.8%, 18.8% and 15.2% respectively (Wang, Wang, & Han, 2013). Traditionally and still today, rural residents never retire and mainly rely on family support and their own labor in old age until they are unable to work anymore. Pension scheme for urban non-employed residents starts in 2007 and cover all urban non-employed residents by 2010. The basic old-age insurance systems for rural and urban residents are integrated in 2014. The replacement rate of low-income, middle-income and high-income urban residents is equal to 75.5%, 62.2% and 50% respectively (Wang, Wang, & Han, 2013). We can see from the above analysis that insufficient pension benefits are the main reason that leads to the phenomena of WRFP.

There are few substantive regulations in the current pension regulatory framework on the condition of working while receiving pension benefits. In China, according to "Labor Contract Law of the People's Republic of China", only after labor concerns are removed or the labor contract is terminated can household agents take up basic old-age pension. However, this regulation cannot be well *enforced in practice for many reasons*. One potential reason is differential access to pension benefits. Only urban residents currently have access to China's state pension system and only about 30 per cent of them are covered by the pension system. Lack of a nationwide basic pension plan that includes both rural and urban residents makes it difficult to manage the public pension. However, launching a new pension insurance system that aims to cover all rural residents and unemployed urbanites implies that WRFP will become more widespread in China, especially in the rural regions of this country. Another reason to expect an increase in WRFP is that workers previously could not transfer their own pension contributions. Only from the start of 2010 has China begun to enable its workers to transfer their retirement benefits or accounts when they move across provinces or find new jobs. The last reason is China's low retirement flexibility. For working agents it is difficult to adjust the pension level. For example, in October, 2011 Shanghai city, which has the highest number of pensioners in China, initiated what is effectively a pilot program that allows urbanites to delay receiving retirement benefits and continue working after retirement age. A report issued by the Chinese Academy of Social Sciences estimated that the pension shortfall is about 1.3 trillion Yuan in 2010, which suggests that WRFP may also exacerbate existing financing problems in the future.

To our knowledge, the potential impact of WRFP is still not clear cut analytically. WRFP has double effects on the social system and individuals. On the one hand, WRFP has negative externalities to pension system. In many countries, inadequate funding, differential pension coverage, fragmented and inconsistent pension system, lack of retirement flexibility and poor management play important roles in stimulating WRFP which in turn reduces efficiency and misuses public pensions relative to the intent of the public pension system. In Russia, working while receiving a pension is a relatively common phenomenon. Low pensionable ages, slim old-age benefits and economic desperation push many elderly to work after retirement. Those working pensioners do not be penalized by any government departments and their pension benefits are fully compatible with wage income (Gerber, & Radl, 2014). Rather than fulfilling original objective of social security, which is to maintain household agents' consumption level at a relatively stable level, WRFP converts the pension program to be a type of extra welfare and well-being payment. The high re-employment rate indicates that the deviation of the pension payment from household agents' desired consumption level is larger than expected. In the long run, WRFP will threaten the fairness of the Pay-As-You-Go (PAYG) pension system. On the other hand, WRFP serves as a hedge against pension risk and provides insurance to most individuals facing health and productivity shocks because of imperfect social security and low income. Since WRFP is an extra instrument to solve the financial problems and provides an option relative to the longevity challenges, WRFP enhances the fiscal sustainability of support system of the elderly and can be considered as the fourth pillar in the overall pension system. In response to the improvement of the social security system, living standards and pension benefits, there has been a decrease in WRFP and a decline in the length of WRFP. For example, according to the Sampling Survey of the Aged Population in Urban/Rural China and National Population Census data, in 1982, 1990, 1995, 2000, 2005and 2010, the labor force participation rates among older people in China are 23.65, 31.36, 29.21, 33.1, 28.2 and 30.48 percent. In 2000, 2005 and 2010, the labor force participation rates in rural and urban areas are 43.2, 36.4, 44.3 percent and 13.4, 9.7, 7.2percent respectively. Thus, WRFP is a double-edged sword and may have ambiguous effects on individual and social welfare.

In this paper we focus on the impacts on household agents' welfare gains caused by WRFP, particularly on the role played by the length of WRFP. There are few papers dealing with WRFP and its impact on the welfare of old workers. Most studies that analyze the pension system do not account for the effect of WRFP on aggregate saving and consumption, and therefore ignore the consequences that it will have on economic growth. On the one hand, WRFP directly affects welfare since it influences how many years household agents stay in the workforce. From another point of view, by determining the length of working life after retirement, WRFP provides a basic incentive to accumulate more for full retirement, which is important for the welfare of old household agents in the long run since savings, causing capital accumulation, are source of economic growth.

Since we don't know of any data that allow us to analyze the effect of WRFP deeply, in this paper we first develop a simple, baseline two-period overlapping generations (OLG) model initially devised by (Diamond, 1965) and a three-period OLG model to consider the economic effects of WRFP on welfare in a utility maximizing framework. In the three-period-OLG model, life is divided into a working period, a WRFP period and a full retirement period. In contrast, in the two-period-OLG model the WRFP period and the full retirement period are combined. In the working period, agents decide how much to consume and how much to save. In the WRFP period, agents decide how much time to spend working while receiving full pension. In the final stage of the life-cycle, agents consume out of their interest income and accumulated savings. The key feature of our models is that we have a WRFP period that combines features of the working period and the full retirement period.

In our two-period-OLG model, we investigate two type of WRFP case: a) fixed length of WRFP, and b) elastic length of WRFP which is determined by the agents' health condition. If the length of WRFP is fixed, the function of WRFP is no different than that of a pension plan; ceteris paribus, WRFP causes young agents to save less and crowds out private capital. When the length of WRFP is elastic, WRFP also distorts young agents' labor supply decision. If the substitution effect of WRFP is dominant, the reduction in young agents' saving reduces the aggregate capital stock. Our results show that the growth effect of WRFP with fixed length is different from that of WRFP with elastic length.

We apply two different models, a two-period-OLG model and a three-period-OLG model, to investigate the welfare effects of WRFP. The difference between the two-period model and the three-period model is that in the three-period-OLG model, representative agents could save some income obtained from the WRFP period for full retirement period. As shown by Aaron (1966) and Samuelson (1975) if a PAYG pension system can provide stationary welfare benefits, it must operate in an inefficient economy with the decline of the capital-labor ratio (Andersen, & Joydeep, 2013). But now, the case for a PAYG pension system with WRFP in an inefficient economy needs to be investigated carefully. We show that, although the existence of WRFP in a PAYG pension system reduces the aggregate capital stock, the change in welfare is determined by the elasticity of capital in the two-period model, whereas the change in welfare depends not only on the elasticity of capital but also on the old-age labor supply in the three-period model.

Theoretical Framework

Agents

We first provide a two-period OLG model in which household agents' income from WRFP period is only used for consumption and not for saving. Household agents who live through two life periods are considered. In the first period t (working period), household agents work and are endowed with one unit of time and one unit of human capital. Time spent in the labor market results in a real wage of w_t. There is a government who implements a mandatory public pension system financed by a Social Security contribution rate τ. The net wage income of the first period $w_t(1-\tau)$ is used for consumption c_t and saving s_t. Thus the period budget constraint is

$$c_t + s_t = (1-\tau)w_t \tag{1}$$

In the second period, *t*+1, the unit time endowment is divided into a fraction λ spent working while receiving a full pension (WRFP period) and a fraction $1-\lambda$ spent fully retired (full retirement period). The WRFP index λ, the length of the WRFP period, measures the relative degree of value that household agents receive full pension benefits while working. In period *t*+1, for the fraction of time supplied on the labor market, household agents receive earnings equal to w_{t+1}. Savings in period *t* become the physical capital in period *t* +1 with an interest rate r_{t+1}.

Under the PAYG Social Security system, old household agents in period *t*+1 receive the total social security income p_{t+1}. Therefore the budget constraint is

$$c_{t+1} = (1+r_{t+1})s_t + \lambda w_{t+1} + p_{t+1} \tag{2}$$

where c_{t+1} represents the consumption in the period *t*+1. That is, the consumption in period *t*+1 is supported by savings plus interest payments from *t* to *t*+1, accrued at the interest rate r_{t+1}, the wage income λw_{t+1} and the pension benefit p_{t+1}.

The difference between Equation (2) and similar constraint in other two-period OLG models is that household agents not only do not need to pay any pension but can also receive pensions while they work in the WRFP period. We emphasize that if old household agents only receive their pension benefits in the full retirement time, the budget constraint is

$$c_{t+1} = (1+r_{t+1})s_t + \lambda w_{t+1} + (1-\lambda)p_{t+1}$$

Household agents' preference function depends on their consumption levels in the two periods and is represented by

$$\overline{U}(t) = u(c_t) + \beta u(c_{t+1}) \tag{3}$$

where β is a factor that reflects time preference. In order to get explicit solutions, we assume the single period utility functions $u(\cdot)$ exhibit logarithmic in consumption so that the substitution and wealth influences of a change in the interest rate cancel one another out.

In an economic equilibrium, taking w_t, w_{t+1}, p_{t+1} and r_{t+1} as given, the agent chooses s_t to maximize the expected lifetime utility, given in Equation (3), subject to the constraints (1) and (2). From the necessary conditions for a maximum we get the optimal saving as follows:

$$s_t = \frac{\beta(1-\tau)}{1+\beta} w_t - \frac{\lambda w_{t+1} + p_{t+1}}{(1+\beta)(1+r_{t+1})} \tag{4}$$

Social Security

The economy is assumed to operate with a PAYG pension system. Specifically, the government levies a proportional social security tax rate $\tau \in (0,1)$ on labor income of the young in the first period. We have the following identity:

$$p_{t+1} = \tau w_t \tag{5}$$

Next, we derive the comparative dynamics effects of changes in λ and τ on s_t. Inserting the expression (5) into (4) yields:

$$s_t = \left(\frac{\beta(1-\tau)}{1+\beta} - \frac{\tau}{(1+\beta)(1+r_{t+1})} \right) w_t - \frac{\lambda w_{t+1}}{(1+\beta)(1+r_{t+1})} \tag{6}$$

Holding the other variables fixed, differentiating Equation (6) with respect to λ and τ yields:

$$\frac{ds_t}{d\lambda} = -\frac{w_{t+1}}{(1+\beta)(1+r_{t+1})} < 0 \tag{7}$$

$$\frac{ds_t}{d\tau} = -\frac{\beta r_{t+1}}{(1+\beta)(1+r_{t+1})} w_t < 0 \tag{8}$$

Equations (7) and (8) show that an exogenous fall in the WRFP index λ, has a nonnegative effect on savings of the young and a rise in the Social Security contribution rate, τ, unambiguously decreases savings of the young. Therefore, WRFP has the same crowding out effect on savings as PAYG pensions.

The Productive Sector

The aggregate labor supply, $1+\lambda$, consists of young agents who in-elastically supply one unit of labor and old agents who spend a fraction λ of time working while receiving a full pension. The production function takes a standard neoclassical constant-returns-to-scale Cobb-Douglas form:

$$Y_t = AK_t^{\alpha}(1+\lambda)^{1-\alpha} \tag{9}$$

where K_t represents the aggregate capital stock, *A* denotes the total factor productivity parameter, and α is the output elasticity of capital (the capital share in production), $0<\alpha<1$. Let $y_t = Y_t/(1+\lambda)$ and $k_t = K_t/(1+\lambda)$ denote output and capital per agent, respectively. Thereby, we obtain output per agent as $y_t = Ak_t^{\alpha}$ Within the framework of a competitive equilibrium, profit maximization leads to the following standard equilibrium results:

$$r_t = \alpha A k_t^{\alpha-1} - 1 \qquad w_t = (1-\alpha)Ak_t^{\alpha} \tag{10}$$

From (10) we obtain

$$\frac{w_t}{1+r_t} = \frac{1-\alpha}{\alpha}k_t \tag{11}$$

The market clearing condition is given by

$$(1+\lambda)k_{t+1} = s_t \tag{12}$$

The Growth Effects of WRFP

The dynamic system is given by (6) and (12).The definition of the steady state is that along the balanced growth path all economic variables are reproduced in identical situations so that the per capita capital stock $k_t = k$ and wage rate $w_{t+1} = w_t = w$ for all *t*.

Proposition 1. A unique non-trivial steady-state per capita capital stock k^* *of the dynamic system exists, and* $\frac{dk^*}{d\lambda} < 0$.

Proof: See Appendix.

First, we should point that it is the steady state capital stock k^* and not the capital stock k that decreases with respect to the WRFP index λ .From the proof of Proposition 1, we can see that k^* is the intersection point of the curve $y=[\alpha(1+\beta)+\tau(1-\alpha)+(1+\alpha\beta)\lambda]k$ and the curve $y=A\alpha\beta(1-\tau)k^{\alpha}$. If the WRFP index λ increases from λ_1 to λ_2 while maintaining all the other parameters constant， labor supply and wages increase accordingly, which leads to the increase of the slope of the first curve. However, since the income from working is used for consumption and not for savings, the capital accumulation does not increase, which means that the second curve keeps invariant. Then from Figure 1, we can see that $k_2^* < k_1^*$. We then reach a conclusion that not only the effect of an extension of WRFP index has a larger effect on the capital accumulation if other parameters are held constant, but also increasing WRFP index will increase the speed of capital accumulation reaching its optimal state.

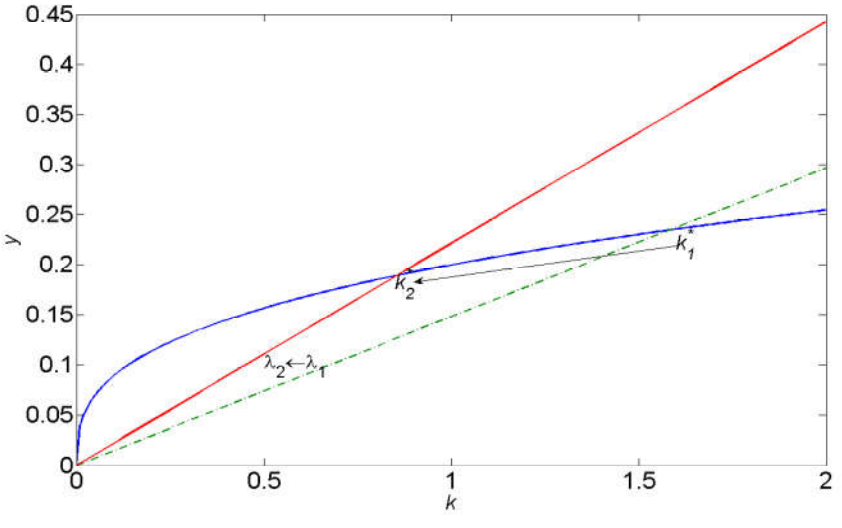

Figure 1. A diagram that shows how the steady state capital stock decreases with respect to the WRFP index. This figure assumes $\alpha=0.35, \beta=0.62, \tau=0.08, \lambda_1=0.1, \lambda_2=0.4$
.

Health Shock and Growth Effects of WRFP

We next turn to the case of the WRFP index which is unfixed and determined by the individual health expenditures. Many preceding studies regard unexpected health status as one of the most important intrinsic motivation that an agent stops working after retirement. As is known, health changes may affect the economic behaviors of household agents through exerting influence on both the ability to work and the productivity of work. Several papers provide important insights into the endogenous relationship between the labor supply by the elderly after retirement and private and public health-care spending (Chakraborty, 2004; Hu, 1979; Leung, & Wang, 2010.] Fanti and Gori (2011) considers how the public provision of health services affects the efficient supply of labor of the elderly in their OLG model.

We assume that the WRFP index, only depends on the individual health status when old and the individual health status, in turn, relies in non-linear form by the size of the health investment h_t . In particular, we assume that the WRFP index is augmented by health investment financed at a balanced budget with a (constant) proportional wage income tax $0 < \eta < 1$ as follows:

$$\lambda = \lambda(h_t) \quad h_t = \eta w_t \tag{13}$$

where $\lambda(0) = \bar{\lambda} > 0$, $\lim_{h \to \infty} \lambda(h) = \tilde{\lambda}$ and $\bar{\lambda} < \tilde{\lambda} < 1$. The relationship between the WRFP index and health expenditure is always assumed to be captured by the generic non-decreasing function, i.e., $\frac{d\lambda(h)}{dh} > 0$, that is, the healthier an old agent, the larger the fraction of WRFP time he supplies labor.

In this case, under the equilibrium condition we have

$$k = \frac{A\alpha\beta(1-\tau-\eta)}{\alpha(1+\beta)+\tau(1-\alpha)+(1+\alpha\beta)\lambda\left(\eta(1-\alpha)Ak^{\alpha}\right)}k^{\alpha} \quad = \frac{B_1}{B_2 + B_3\lambda\left(B_4k^{\alpha}\right)}k^{\alpha} \tag{14}$$

where $B_1 = A\alpha\beta(1-\tau-\eta)$, $B_2 = \alpha(1+\beta)+\tau(1-\alpha)$, $B_3 = 1+\alpha\beta$ and $B_4 = \eta(1-\alpha)A$.

Proposition 2. A non-trivial steady-state per capita capital stock k^* of the dynamic system described by Equation (14) exists. If $\lambda'(h) > 0$, $\frac{\partial k^*}{\partial \tau} < 0$.

Proof: See Appendix.

We now discuss the economic mechanism behind Proposition 2 which concludes that if $\lambda = \lambda(h)$ is a non-decreasing function of *h*, the steady state per capita capital stock k^* decreases with respect to the contribution rate τ in the short run. First, notice that the WRFP index depends only on the household agent's health status after retirement. Increasing social security contribution rate causes negative influence on capital stock and wages, and this in turn reduces health expenditure to the extent that pension benefits crowd out savings. Small reduction in health status would result in strong reduction in the ability to work. The development of Chinese pension system indicates that when the social security contribution rate is about the 0(absence of public pensions), 8 and 15 percent of wage income， the WRFP index is about 82, 64 and 15 per cent of their second period of time endowment. Thus, a brief verbal summary of the results is that the larger(smaller) the contribution rate is, the smaller (larger)the capital stock is, thereby the larger (smaller)the interest rate is, and, therefore, the lower (higher)the incentives to increase health expenditure is. This negative effect on the (neoclassical) economic growth is due to channel of crowding out effect of WRFP on savings and capital accumulation as we have stated earlier.

We also find that the growth effect of WRFP is familiar with that of the Social Security contribution rate. They both have negative effects on k^*. The intuition is straightforward. If household agents contribute more in their productive working life, they can work less in their retired years, or else they have to work longer to maintain the same living standard. Thus WRFP is unavoidable and has substitution effect for pension benefit in a low level pension system.

Numerical Examples

We now examine the comparative static analytic solutions of Proposition 2 more intuitively with some numerical examples. We suggest two functional forms for the WRFP index. The first functional form of WRFP index used in this paper is proposed by (Blackburn, Pietro, & Blackburn, 2002) and is also adopted by (Fanti, & Gori, 2011). Their functional form is

$$\lambda(h_t) = \frac{\bar{\lambda} + \tilde{\lambda} h_t^{\sigma}}{1 + h_t^{\sigma}} \tag{15}$$

where $\bar{\lambda} < \tilde{\lambda}$ and $\bar{\lambda}$ represents the natural health status of an agent when there is no health spending. One can interpret $\sigma > 0$ as the efficiency of health spending.

Since $\frac{d\lambda(h_t)}{dh_t} = \frac{\sigma h_t^{\sigma-1}(\tilde{\lambda} - \bar{\lambda})}{1 + h_t^{\sigma}} > 0$, the right hand side of (15) is an increasing function of h_t , which means that there exists a unique, non-trivial globally stable steady state.

Figure 2 depicts the relationship between the two sides of Equation (A.4) (The equation is given in the Appendix) and k under alternative values of τ . The graph corresponds to the parameter values τ =0.05, 0.1, 0.15, 0.2, 0.25, 0.3.The scale parameter A is fixed at 1. Regarding preferences, the discount factor β is assumed to be equal to 0.60.The other parameters chosen to plot the two figures are as follows: $\bar{\lambda} = 0.2$, $\tilde{\lambda} = 0.9$, $\eta = 0.12$. For the elasticity of capital, we follow (Kraay, & Raddatz, 2007) and set α to 0.35. We can see from Figure 2 that the described structure of steady-state capital per agent is consistent with Proposition 2.

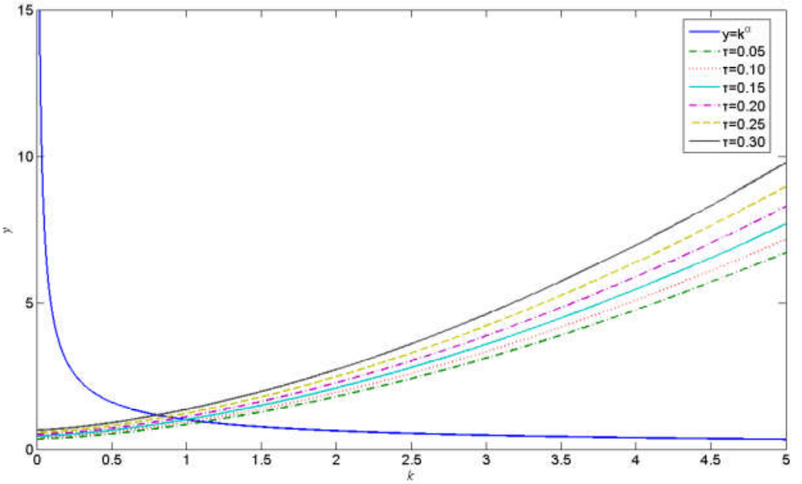

Figure 2. The steady state of k with τ=0.05,0.1,0.15,0.2,0.25,0.3 (α=0.35,σ=0.6) , where the horizontal axes refers to k and the vertical axes refers to the two sides of Equation (A.4).*

Consistent with the assumption of working time after retirement in (Gori, L. & Sodini, 2011), we assume the second functional form of the WRFP index as follows:

$$\lambda(h_t) = (\bar{\lambda} + \tilde{\lambda} h_t)^{\delta} \tag{16}$$

$\frac{d\lambda(h_t)}{dh_t} > 0$ means that there is a unique stable steady state. If δ satisfies $0 < \delta < 1$, $\lambda(h_t)$ is a concave function, indicating that raising health expenditure determines a less than proportional increase in the person's state of health.

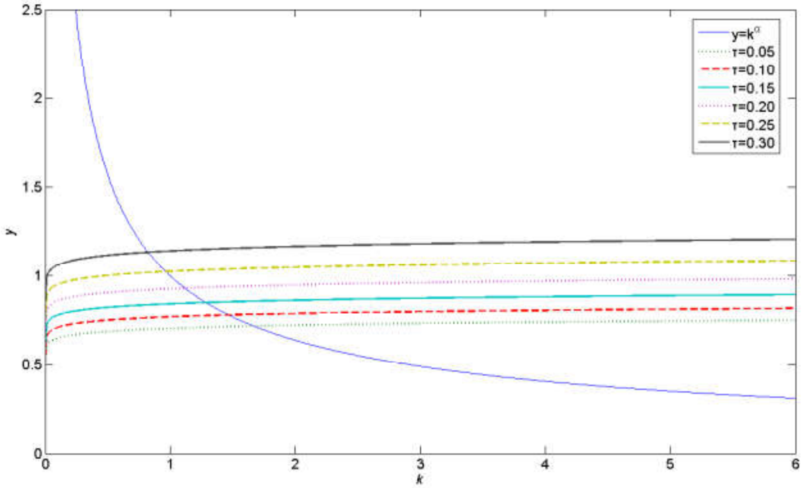

*Figure 3. The steady state of k*withτ=0.05,0.1,0.15,0.2,0.25,0.3 (α=0.35,δ=0.6), where the horizontal axes refers to k and the vertical axes refers to the two sides of Equation (A.4).*

Figure 3 shows how the features of a steady state change when τ varies. Specifically, we consider τ =0.05, 0.1, 0.15, 0.2, 0.25, 0.3. We observe that the curves intersect at a single point, giving a unique steady state. Figure 3 also shows that a larger value of δ leads to a lower per capita capital stock.

From the comparative static analysis we find that the long-run growth effects of WRFP are negative which means that a higher WRFP index reduce the steady-state capital intensity. A higher WRFP index has two reinforcing effects on k^* . Intuitively, although a higher WRFP index increases the labor supply, a higher WRFP index also increases the amount of aggregate investments necessary to support a given steady-state k^* . Given savings per unit of labor supply this reduces capital intensity. Since the steady stats in Propositions 1 and 2 are locally stable, if the reduction of savings per unit of efficient labor occurs in a locally stable steady state, the value of steady-state capital intensity must fall.

The Welfare Effects of WRFP

In this subsection we investigate how the WRFP index influences the steady-state welfare level. We calculate welfare gains in terms of equivalent variations in the utility of consumption in a steady state framework.

The first-period and second-period consumption can be rewritten as

$$c_t = (1-\tau)w_t - s_t = \frac{1}{1+\beta}\left[(1-\tau)w_t + \frac{\tau w_t + \lambda w_{t+1}}{1+r_{t+1}}\right] \tag{17}$$

$$c_{t+1} = (1+r_{t+1})\left(\frac{\beta(1-\tau)}{1+\beta}w_t - \frac{\lambda w_{t+1} + \tau w_t}{(1+r_{t+1})(1+\beta)}\right) + \lambda w_{t+1} + \tau w_t$$

$$= \frac{\beta(1+r_{t+1})}{1+\beta}\left((1-\tau)w_t + \frac{\lambda w_{t+1} + \tau w_t}{(1+r_{t+1})(1+\beta)}\right) \quad = \beta(1+r_{t+1})c_t \tag{18}$$

By substituting (17) and (18) into (3), in equilibrium, the lifetime utility of the agent can be expressed as

$$\bar{U}_t = \beta \ln \beta + (1+\beta)\ln c_t + \beta \ln(1+r_{t+1}) \tag{19}$$

Since $k_t = K_t / (1+\lambda)$, at the steady state, we obtain

$$\frac{dk_t}{d\lambda} = -\frac{k_t}{1+\lambda} \tag{20}$$

From Equation (10) we have

$$\frac{dr_t}{d\lambda} = \frac{dr_t}{dk_t}\frac{dk_t}{d\lambda} \quad = -A\alpha(\alpha-1)k_t^{\alpha-2}\frac{k_t}{1+\lambda} \tag{21}$$

$$= -(\alpha-1)A\alpha k_t^{\alpha-1}\frac{1}{1+\lambda} \quad = -(\alpha-1)\frac{1+r_t}{1+\lambda}$$

$$\frac{dw_t}{d\lambda} = \frac{dw_t}{dk_t}\frac{dk_t}{d\lambda} \quad = -A\alpha(1-\alpha)k_t^{\alpha-1}\frac{k_t}{1+\lambda}$$

$$= -A\alpha(1-\alpha)k_t^{\alpha}\frac{1}{1+\lambda} = -\alpha\frac{w_t}{1+\lambda} \tag{22}$$

Ignoring the time subscripts and substituting Equation (11) into Equation (17), together with the steady state conditions and the relationship

$$\frac{w}{k} = (1-\alpha)Ak^{\alpha-1} = \frac{1-\alpha}{\alpha}(1+r)$$

we have

$$c_t = c = \frac{1}{1+\beta}\left[(1-\tau)w + (\tau+\lambda)\frac{1-\alpha}{\alpha}k\right] = \frac{\alpha(1-\tau)w + (1-\alpha)(\tau+\lambda)k}{\alpha(1+\beta)} \tag{23}$$

Substituting (23) into (19), at the steady state, we have the indirect utility function as follows:

$$\bar{U}(\lambda) = \beta\ln\beta + (1+\beta)\ln c + \beta\ln(1+r)$$

Proposition 3. There exists a unique α^* that satisfies

$$\frac{d\bar{U}(\lambda)}{d\lambda} < 0 \quad \textit{if } \alpha > \alpha^*$$

and

$$\frac{d\bar{U}(\lambda)}{d\lambda} > 0 \quad \textit{if } \alpha < \alpha^*$$

Proof: See Appendix.

Proposition 3 states that the effect of WRFP on welfare gains in the long run is ambiguous. We show that there does not exit a socially optimal WRFP state where the household agents' indirect lifetime utility of is maximized. This result is a consequence of two counteracting changes taking place as the output elasticity of capital is altered. For example, an increased WRFP index, in itself, tends to increase lifetime labor supply and thereby income. However, for higher values of the WRFP index, the reaction of effective labor supply is not so strong and a further increasing WRFP index makes household agents choose to save not so more. If the output elasticity of capital α is large enough and exceeds a certain threshold value α^*, the output elasticity of labor or the output elasticity of λ will be too small and increasing the WRFP index will decrease welfare. When the output elasticity of capital α is too small, on the contrary, increasing the WRFP index will increase welfare.

The Critical Points of Output Elasticity of Capital in the Evolution of Welfare Effects of WRFP

As stated in the previous subsection, the signs of the derivatives of the indirect utility with respect to the WRFP index are ambiguous. For these cases, it is of some interest to illustrate the sources for this ambiguity and investigate how the welfare evolves with changes in the output elasticity of capital. In the remainder of this subsection the value of α^* is then studied in the condition of $\frac{d\bar{U}(\lambda)}{d\lambda}>0$ or $\frac{d\bar{U}(\lambda)}{d\lambda}<0$.

For notational simplicity, let $\theta=\frac{\beta}{1+\beta}$. Since α satisfying $\frac{d\bar{U}(\lambda)}{d\lambda}>0$ for all $\lambda\in(0,1)$ is such that $g(\alpha)>0$, where the function $g(\alpha)$ is defined in (A.8) with

$$g(\alpha)=\frac{-\alpha(1-\tau)(1+r)+(1-\tau)}{(1-\tau)(1+r)+(\tau+\lambda)}+\frac{\beta}{1+\beta}(1-\alpha)$$

and

$$\frac{d\bar{U}(\lambda)}{d\lambda}=\frac{1+\beta}{1+\lambda}g(\alpha)$$

Simple algebra shows that the following inequality holds:

$$-\alpha(1-\tau)(1+r)+(1-\tau)+\theta(1-\alpha)(1-\tau)(1+r)+\theta\tau(1-\alpha)+\lambda\theta(1-\alpha)>0 \quad (24)$$

for all $\lambda \in (0,1)$.

Since $\theta(1-\alpha)>0$ and the left hand side of (24) is a linear function of λ, the left hand side of (24) are bigger than 0 at $\lambda=0$. That means the following inequality holds:

$$-\alpha(1-\tau)(1+r)+(1-\tau)+\theta(1-\alpha)(1-\tau)(1+r)+\theta\tau(1-\alpha)>0$$

Then if the inequality $\frac{d\bar{U}(\lambda)}{d\lambda}$>0 holds for all $\lambda \in (0,1)$, the value α^* are chosen so that

$$\left[(1+\theta)(1+r)-(1+r+\theta r)\tau\right]\alpha^*-(1-\tau)(1+\theta-\theta r)<0 \quad (25)$$

Another case can be tackled in a similar way. If the inequality $\frac{d\bar{U}(\lambda)}{d\lambda}$<0 holds for all $\lambda \in (0,1)$, then the inequality $g(\alpha)<0$ holds for all $\lambda \in (0,1)$. That is

$$-\alpha(1-\tau)(1+r)+(1-\tau)+\theta(1-\alpha)(1-\tau)(1+r)+\theta\tau(1-\alpha)+\lambda\theta(1-\alpha)<0 \quad (26)$$

for all $\lambda \in (0,1)$.

Since $\theta(1-\alpha)>0$, the left hand side of (26) should be less than 0 at $\lambda=1$. That is, the following inequality holds:

$$-\alpha(1-\tau)(1+r)+(1-\tau)+\theta(1-\alpha)(1-\tau)(1+r)+\theta\tau(1-\alpha)+\theta(1-\alpha)<0$$

Then if inequality $\frac{d\bar{U}(\lambda)}{d\lambda}$<0 holds for all $\lambda \in (0,1)$, α^* satisfies

$$\left[\theta+(1+\theta)(1+r)-(1+r+\theta r)\tau\right]\alpha^*-\theta-(1-\tau)(1+\theta-\theta r)>0 \quad (27)$$

Theoretically, based on the Equations (25) and (27), the range of α is divided into three regions. In the first region, since $\frac{d\bar{U}(\lambda)}{d\lambda}$ >0, so that welfare increases with WRFP index. However, in the second region ($\frac{d\bar{U}(\lambda)}{d\lambda}$ <0), an increase in the WRFP index decreases the welfare. Finally, in the third one, due to the sign of $\frac{d\bar{U}(\lambda)}{d\lambda}$ is ambiguous, the relationship between the welfare and WRFP index is not clear.

Figure 4 illustrates the relationship between the left sides of (25) and (27) and α with the parameters as follows: τ =0.32, β =0.6 and *r*=0.05. The slopes and intercepts of the two lines are 0.8784, -1.2972 and1.844, -2.0362 corresponding to $\frac{d\bar{U}(\lambda)}{d\lambda}$ >0 and $\frac{d\bar{U}(\lambda)}{d\lambda}$ <0. Figure 4 shows that for our parameter settings we only obtain the case of $\frac{d\bar{U}(\lambda)}{d\lambda}$ >0 for $\alpha \in (0.8784,1)$ (the shaded area), that is, the welfare gains only increase with the elasticity of capital.

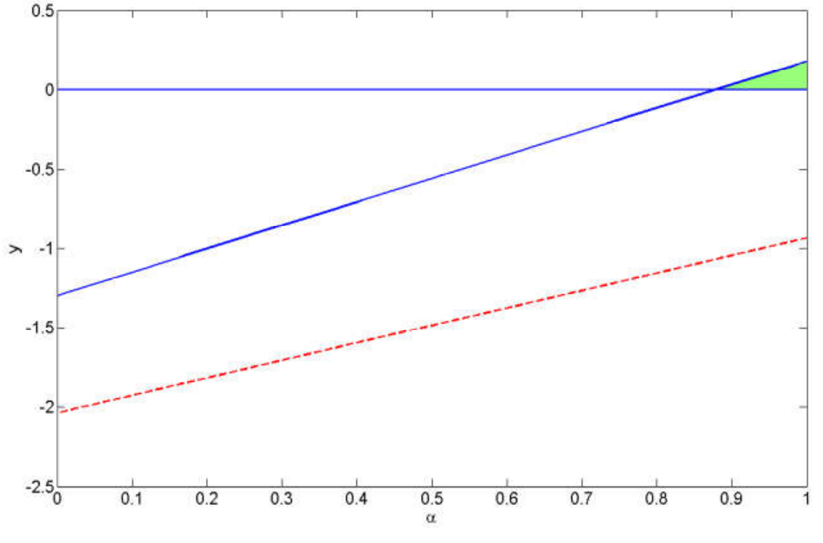

Figure 4. The welfare gains and the areas of elasticity of capital. This figure shows the relationship between the left sides of (25) (solid line) and (27) (dash line) (vertical axes) and α(horizontal axes)with the parameters as follows: τ=0.08,β=0.62 and r=0.03.

The Welfare Effects of WRFP in a Three Period Model

The Model

The proposed two-period OLG model is tractable but perhaps not realistic and fails to capture many relevant aspects. Our stylized two-period OLG model cannot represent a WRFP period in which pensions are received and work may be chosen distinctly from a period in which, due to age, WFRP is not a viable choice for a household agent. Therefore, it may be more appropriate to consider an OLG model consisting of three periods rather than an OLG model consisting of two periods. Another shortcoming of the two-period model is that we find the representative agent will save in the WRFP period in the three-period model, which also occurs in practice and which cannot be captured in a two-period OLG model. WRFP not only enables the agents to save more due to earning a higher income in the WRFP period but also affects aggregate savings and labor supply which results in a higher capital-labor ratio. Our results show that welfare gains depend not only on the elasticity of capital but also on the length of the WRFP period for the proposed three-period OLG model.

We now suppose that household agent live for three periods. As in the two-period OLG model, in period *t*, young-adult household agents work for wage rate w_t, consume and save an amount of c_t and s_t. Thus we have the same budget constraint as (1).

In period *t*+1, household agents receive earnings equal to w_{t+1} for the fraction of time supplied on the labor market. Under the PAYG Social Security system, old household agents in period *t*+1 and *t*+2 receive the total social security income p_{t+1}. They are entitled to an amount of pension benefits λp_{t+1} in the period *t*+1, which is proportional to the length of period *t*+1. Therefore the budget constraint is

$$c_{t+1} + s_{t+1} = \lambda w_{t+1} + (1 + \lambda r_t) s_t + \lambda p_{t+1} \tag{28}$$

where c_{t+1} and s_{t+1} represent consumption and saving in the period *t*+1. That is, the consumption and saving during the period *t*+1 is supported by savings plus interest from *t* to *t*+1, accrued at the interest rate r_t, the wage income λw_{t+1} and the pension benefit λp_{t+1} for the WRFP period. Here s_{t+1} is the final savings including the saving in period *t*. Since the length of period *t*+1 is λ, the interest is $(1 + \lambda r_t) s_t$.

In period *t*+2, household agents live on the basis of (1) the amount of resources saved during the period *t*+1 plus the accrued interest $(1-\lambda) r_{t+1}$ which are spent in old age when agents are retired, and (2) the public pension benefits $(1-\lambda)p_{t+1}$. The individual budget constrain is

$$c_{t+2} = (1+(1-\lambda)r_{t+1})s_{t+1} + (1-\lambda)p_{t+1} \tag{29}$$

where c_{t+2} is the consumption at time $t + 2$.

Household agents' utility function depends on their consumptions in the three periods:

$$\widetilde{U}(t) = u(c_t) + \beta(\mu u(c_{t+1}) + (1-\mu)u(c_{t+2})) \tag{30}$$

where $\mu \in [0,1]$ indicates relative importance of WRFP. We continue to assume the single period utility functions $u(\cdot)$ exhibit logarithmic utility.

Given budget constraints (1), (28) and (29), agents choose their consumptions and savings to maximize lifetime utility $\widetilde{U}(t)$. Solving the resulting constrained optimization problem with respect to the optimal saving rate yields the optimality results for household agents:

$$s_t = \frac{\beta}{1+\beta}(1-\tau)w_t - \frac{\lambda}{1+\beta}\frac{w_{t+1}}{1+\lambda r_t} - \frac{\lambda}{1+\beta}\frac{p_{t+1}}{1+\lambda r_t} - \frac{(1-\lambda)p_{t+1}}{(1+\beta)(1+\lambda r_t)(1+(1-\lambda)r_{t+1})} \tag{31}$$

$$s_{t+1} = \frac{\beta}{1+\beta}(1-\mu)(1-\tau)(1+\lambda r_t)w_t + \frac{\beta}{1+\beta} \times \lambda(1-\mu)w_{t+1} + \frac{\beta}{1+\beta}\lambda(1-\mu)p_{t+1} - \frac{1+\beta\mu}{1+\beta}\frac{(1-\lambda)p_{t+1}}{1+(1-\lambda)r_{t+1}} \tag{32}$$

By taking the derivative of Equations (31) and (32) with respect to λ, it is easy to show that

$$\frac{\partial s_t}{\partial \lambda} = \frac{-1}{1+\beta}\left(\frac{w_{t+1}}{(1+\lambda r_t)^2} + \frac{p_{t+1}}{(1+\lambda r_t)^2}\right.$$

$$\left. + \frac{p_{t+1}(1+r_t+(1-\lambda)^2 r_t r_{t+1}}{(1+\beta)\left((1+\lambda r_t)(1+(1-\lambda)r_{t+1})\right)^2}\right) < 0 \tag{33}$$

$$\frac{\partial s_{t+1}}{\partial \lambda} = \frac{\beta}{1+\beta}(1-\mu)(1-\tau)r_t w_t \quad + \frac{\beta}{1+\beta}(1-\mu)w_{t+1}$$

$$+\frac{\beta}{1+\beta}(1-\mu)p_{t+1} \quad + \frac{1+\beta\mu}{1+\beta}\frac{p_{t+1}}{(1+(1-\lambda)r_{t+1})^2} > 0 \tag{34}$$

As can be observed in (33) and (34), s_t is a decreasing function of λ and s_{t+1} is an increasing function of λ. That is, an increase of the WRFP index would have negative effect on savings in the working period and have positive effect on final savings. Therefore WRFP has crowding out effect in the working period and crowding in effect on savings in the WRFP period. When household agents are young, they will reduce saving because of they anticipate they can work even when they are old. However, when household agents are old, they find their saving level is too low to maintain a normal life and they will increase working time and save more.

The market clearing condition is given by

$$(1+\lambda)k_{t+2} = s_{t+1} \tag{35}$$

In equilibrium, consumption in the three periods can be rewritten as

$$c_t = \frac{1}{(1+\beta)(1+\lambda r_t)} \left((1-\tau)(1+\lambda r_t)w_t + \lambda w_{t+1} + \lambda p_{t+1} + \frac{(1-\lambda)p_{t+1}}{1+(1-\lambda)r_{t+1}} \right) \quad (36)$$

$$c_{t+1} = \frac{\beta\mu}{1+\beta} \left((1-\tau)(1+\lambda r_t)w_t + \lambda w_{t+1} + \lambda p_{t+1} + \frac{(1-\lambda)p_{t+1}}{1+(1-\lambda)r_{t+1}} \right) \quad (37)$$

$$c_{t+2} = \frac{\beta(1-\mu)(1+(1-\lambda)r_{t+1})}{(1+\beta)} \quad ((1-\tau)(1+\lambda r_t)w_t$$

$$+\lambda w_{t+1} \quad +\lambda p_{t+1} + \frac{(1-\lambda)p_{t+1}}{1+(1-\lambda)r_{t+1}}) \quad (38)$$

Rewrite (37) and (38) as follows:

$$c_{t+1} \quad = \beta\mu(1+\lambda r_t)c_t \quad (39)$$

$$c_{t+2} \quad = \beta(1-\mu)(1+\lambda r_t) \quad (1+(1-\lambda)r_{t+1})c_t \quad (40)$$

Using budget constraint (1) and market clearing condition, we get

$$c_t = (1-\tau)w_t - s_t = (1-\tau)w_t - (1+\lambda)k_{t+1}$$

As usual we may define steady-state equilibrium as an equilibrium sequence of constant wages, output and capital per agent, and consumption pairs. We omit all the indexes for notational convenience. Together with expressions (39) and (40), we have the maximized indirect utility function, in steady state,

$$\widetilde{U}(\lambda) = C + \beta\ln(1+r\lambda) \quad +\beta(1-\mu)\ln(1+(1-\lambda)r)$$

$$+(1+\beta)\ln((1-\tau)w - k(1+\lambda)) \quad (41)$$

where C is a constant with respect to λ.

The Welfare Effects of WRFP

Household agents' optimization problem is to maximize $\widetilde{U}(\lambda)$. The first order condition for the optimal WRFP index is given by

$$\frac{d\widetilde{U}(\lambda)}{d\lambda} = \frac{\beta}{1+r\lambda}\left(r+\lambda\frac{dr}{d\lambda}\right) + \frac{\beta(1-\mu)}{1+(1-\lambda)r}\left((1-\lambda)\frac{dr}{d\lambda}-r\right)$$

$$+\frac{(1+\beta)}{(1-\tau)w-(1+\lambda)k}\left((1-\tau)\frac{dw}{d\lambda} - k-(1+\lambda)\frac{dk}{d\lambda}\right) \tag{42}$$

We can rewrite (42) as

$$\frac{d\widetilde{U}(\lambda)}{d\lambda} = \frac{\beta}{1+r\lambda}\left(r+\frac{\lambda(1-\alpha)(1+r)}{1+\lambda}\right) - \frac{\beta(1-\mu)}{1+(1-\lambda)r}\left(\frac{(1-\lambda)(1-\alpha)(1+r)}{1+\lambda}+r\right)$$

$$+\frac{\alpha(1+\beta)(1-\tau)(1-\alpha)(1+r)}{(1+\lambda)(\alpha(1+\lambda)-(1-\tau)(1-\alpha)(1+r))} \tag{43}$$

Proposition 4. If $\lambda > \frac{1}{2}$ and $\alpha > \frac{1}{2}$, $\frac{d\widetilde{U}(\lambda)}{d\lambda} > 0$ holds.

Proof. See Appendix.

We now turn to the welfare effects and growth impacts of Proposition 4. It is well known that welfare rationale is impossible for the traditional PAYG scheme if the economy is dynamically efficient in a standard OLG model with exogenous labor supply. In other words, welfare effects exist if the initial steady state is dynamically inefficient. Our results show that the welfare effects are ambiguous in an economy with WRFP phenomena.

As a mechanism for consumption smoothing and a means of insurance, WRFP can surmount the PAYG challenge. The numerical calculation suggests that household agents tend to work for a longer time and therefore the length of the residual full retirement period is lowered. That is, increasing WRFP index may yield welfare gains. The positive effect on the welfare at the individual level stems from two channels. On the one hand, a change of the WRFP index affects the labor force and hence the capital-labor ratio. A higher WRFP index means a higher elasticity of the labor force which makes the capital-labor ratio fall and produces a higher return on savings, as reflected in more consumption available to full retirees. On the other hand, an increase of the WRFP index not only increases the working period but also increases the pension benefits, which will unambiguously improve welfare levels.

However, if we connect WRFP to distortions in the savings, WRFP will increase unemployment and reduce capital per capita and thus in generally reduce wages. As a result, welfare impairment of WRFP occurs. As illustrated by Proposition 3, lifetime welfare at the steady-state has different monotonic characteristics with respect to WRFP index depending on whether the elasticity of capital is above or below a critical value α^*. For low levels of the elasticity of capital that coincide with a higher WRFP index, household agents would consume more in the working time period and therefore have a higher lifetime welfare, whereas the opposite holds when the share of capital share in production is high. Furthermore, from Proposition 4, we know that if the WRFP index and the elasticity of capital is sufficiently large to satisfy $\lambda > \frac{1}{2}$ and $\alpha > \frac{1}{2}$, $\widetilde{U}(\lambda)$ is increasing with respect to λ. That is, a higher WRFP index improves steady-state welfare, which indicates that there is no optimal welfare in the steady state condition for household agents' problem.

Calibration

According to the analytical results described earlier, when $\frac{d\widetilde{U}(\lambda)}{d\lambda} > 0$ holds, we need the condition of $\lambda > \frac{1}{2}$ and $\alpha > \frac{1}{2}$. Thus it is interesting to investigate what's about $\lambda \le \frac{1}{2}$ or $\alpha \le \frac{1}{2}$? Figure 5 provides the change features of $\frac{d\widetilde{U}(\lambda)}{d\lambda}$ with respect to WRFP index. We find that for a large range of WRFP index, $\frac{d\widetilde{U}(\lambda)}{d\lambda} > 0$ holds. Furthermore, it is not a typical economy setting in many countries that the share of capital share in production is bigger than 1/2. Therefore, it is necessary to study the optimal welfare in the steady state condition by calibration.

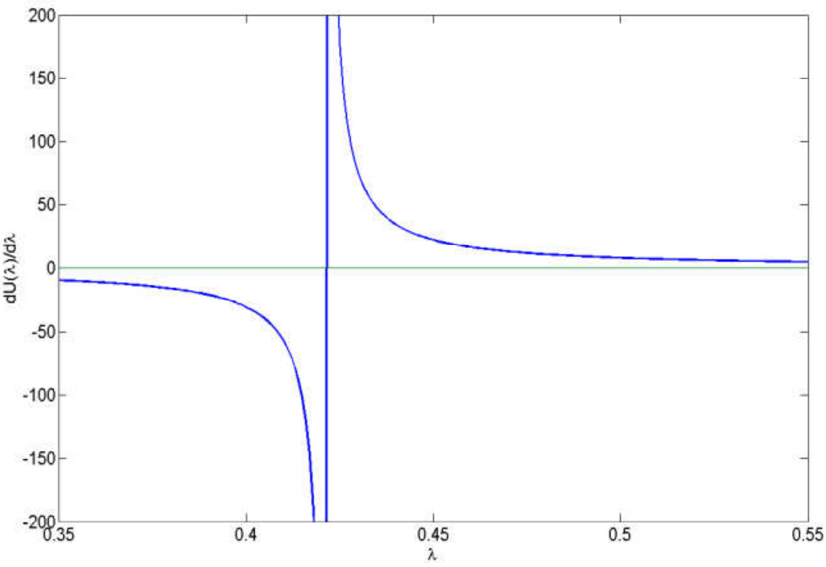

Figure 5. A diagram shows that not for all WRFP index, holds. This figure assumes $\alpha = 0.35, \beta = 0.62, \tau = 0.08, r = 0.03, \mu = 0.05$.

Parameterization

Given economic settings for the technologies, preference and pension system, a steady state competitive equilibrium is such that:(1) households make optimal consumption and the WRFP index decisions by solving the utility maximization problem in (30) and (41);(2) standard equilibrium results in (10) by solving the profit maximization problem;(3) the market clearing condition(35).

To calibrate the model numerically, values to the parameters of preferences and technologies are either selected from related literature or matched actual economic settings in the Chinese rural and urban economy around the period 2009 to 2016 when pension system is adopted only for a few years and levels of pension benefits are low for the rural residents. The wage and interest rate of the benchmark economy are set to be equal to equilibrium levels in the steady state of the economy where there isn't any pension system. Hence, the year 2009 is assumed to be the benchmark year for our economic calculations. Thus we can investigate the impact of WRFP while abstracting from its negative externalities on pension system. We then focus on the pecuniary effects of the WRFP on household agents' welfare by studying the general equilibrium with endogenous factor prices. On the balanced growth path, we suppose the economy reaches its new steady-state equilibrium within 30 periods. The balanced growth path are created according to the following route： from without pension benefits before 2009 to low pension benefits in rural areas and finally to high pension benefits in urban areas between 2009 and 2016. Two scenarios of the actual economic environment in the Chinese rural economy are considered: (1) low pension benefit level and high WRFP index with τ=0.05，λ=0.40 (LH scenario). (2) high pension benefit level and low WRFP index with τ=0.20，λ=0.10 (HL scenario).We also provide transition paths of the general equilibrium state.

The value of the subjective rate for the time preference factor, β , is set equal to 0.62 on an annual basis so that the annual risk-free real interest rate in steady state is approximately 2.26%, the average ex-post real interest rate for the period 2009 to 2016. The output elasticity of capital in the production function is usually to be estimated as 0.3. The labor in Chinese rural is comparatively cheaper, thus, the output elasticity of income is lower, while the output elasticity of capital is higher. Hence, it is proper to assume that α in China is 0.35. In order to focus on the effects of WRFP changes, technological progress is left aside so that the total factor productivity parameter A can be normalized as 1. These values are baseline values of the parameters. A standard algorithm is adopted to find the steady state equilibrium for the three hypostasized settings and the transition paths by solving for the optimal conditions and the market equilibrium conditions. The algorithm first chooses initial values for some endogenous variables and then updates them by iterating between the productions, household agents and pension system until convergence.

Steady-state results

The values of key model variables of the steady-state results for the LH scenario, HL scenario and the general equilibrium results are presented in Table 1.

	LH scenario	HL scenario	General equilibrium
Wage rate			
Working period	0.3114	0.4210	0.3025
WRFP period	0.2708	0.3542	0.2676
Interest rate	2.23%	2.23%	2.35%
Consumption			
Working period	0.6933	0.7135	0.7361
WRFP period	0.7389	0.7865	0.8012
Full retirement period	0.9856	1.2463	1.3331
Saving			
Working period	0.5428	0.5012	0.5127
WRFP period	0.6321	0.6173	0.6203
Capital accumulation	0.3542	0.3423	0.3401
Welfare level	-20.1303	-16.2526	-18.6632

Table 1. The steady-state values of key model variables.

The wage rate of HL scenario is higher than that of LH scenario and the wage rate of general equilibrium case is between them. The wage rate in working period is higher than that in WRFP period. The increases of long run wage rate in working period and WRFP period are 37% and 41.2% respectively. The reason lies in that a lower WRFP index will make labor relatively scarcer and will decrease labor supply which makes a decline in wage and an increase in interest rates. If the labor supply is inelastic, wage rates will increase.

Consumption ranking in an ascending order is: LH scenario, HL scenario and general equilibrium. Consumptions in working period, WRFP period and full retirement period show the same gradually increasing tendency. A change in the WRFP index affects the allocation of consumption between the working period, the WRFP period and the full retirement period and thus changes the equilibrium real interest rates. The saving has familiar features as the wage rate. The saving of HL scenario is higher than that of LH scenario， and the saving of general equilibrium case is intermediate. The saving in working period is lower than that in WRFP period. Their interpretation is fairly straightforward. WRFP means household agents choose to work longer and thus get a higher income level during WRFP period, implying that household agents can save more to offset low lifetime income caused by low pension benefits. Rising savings may lead to increasing capital stock and then decreasing marginal product of capital, the interest rate. Finally, a higher real interest rate increases the consumption of the WRFP period and full retirement period without affecting the savings of the working period.

As to the effects of WRFP on capital accumulation, LH scenario is the biggest among the three scenarios since WRFP is more important for a household in rural areas than that in urban areas. Increasing the WRFP index increases total effective labor supply and, thus, decreases the capital accumulation. However there are no significant different effects of WRFP on capital accumulation for the three scenarios because saving from WRFP period is not the main source of total saving (since we suppose $\lambda \leq 0.5$). The average welfare gains are negative for the three economic settings, and HL scenario is the largest among the three cases. As a consequence, although WRFP has an incentive effect on household agents, welfare losses arisen from its negative externalities exceed welfare gains in the total.

Transition paths

Under the assumption that adjustment to the new steady state takes place in 30 years and factor prices are endogenous, we analyze the long-run impact of WRFP on key variables and present their transition paths of key variables in Figure 6.

As exhibited by Figure 6, different degrees of WRFP affect the transition paths of different variables and the long run levels of per capita quantities. Figure 6 shows that an increase in WRFP leads to an immediate decrease in the wage rate.

The figure shows that the wage rates in working period and WRFP period decrease by about 0.70 and 0.71 percentage points annually. Thus household agents face negative wage rate-related WRFP effect. Note that from time=2 onward to time=5, the real interest rate decreases and then increases even more and finally reaches to its steady state. Figure 6 also shows that the interest rate rises from 1.85% to 2.25%, which is consistent with previous studies. This is the positive interest rate-related WRFP effect. The capital accumulation increases from time 1 to time 3, reaches a peak at time 4 and then decreases thereafter. After 14 years, capital accumulation convergences to its new steady state. Then we have a negative capital accumulation-related WRFP effect. There is no any effect that plays a leading role at the beginning and we see an increase in labor supply lead to a fluctuation in welfare gains for about 10 years. After that and we see stable transition paths of wage rate, interest rate and capital accumulation. Since negative effects now dominate and we see welfare gains begin to settle down.

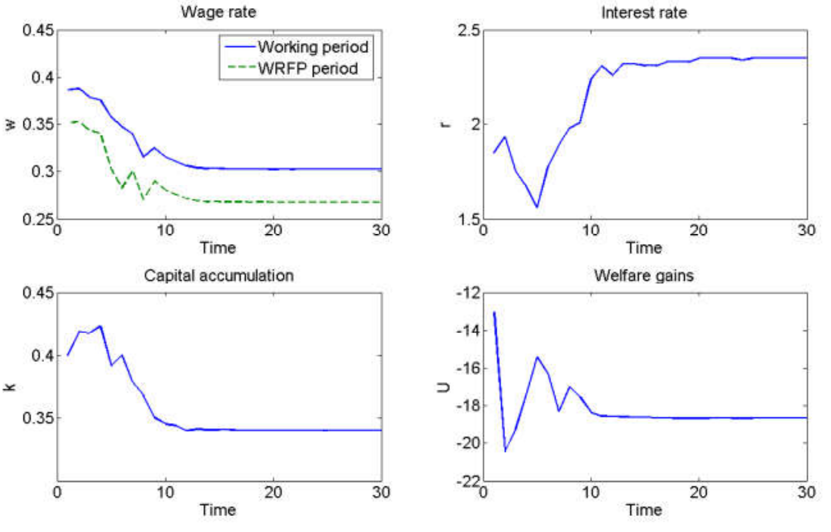

Figure 6. Transition paths after WRFP shock. Here t=0 corresponds to the absence of a Social Security system before 2009. At t=1, changes of the WRFP index occur, which corresponds to the year 2009.

Conclusion

In this paper we investigate a special phenomenon of WRFP available to household agents in some countries. Although we introduce our problems from Chinese pension environments, we investigate the economic effects of WRFP outside the context of any particular country like China in order to provide a systematic way of investigating WRFP.

We show that WRFP matters both for the growth performance and for welfare gains. We explore the economic consequences and welfare implications of WRFP in a two-period OLG model and a three-period OLG model that differ in whether WRFP occurs in a separate period. In our two-period OLG model, we discuss the existence and uniqueness of non-trivial steady states which guarantee global stability for comparative-static exercises. In the steady state, an exogenous increase of the amount of the old-age labor supply and Social Security contribution rate would have a negative effect on output per agent. In the model with endogenous old-age labor supply, we suppose the old-age labor supply is determined by the individual's health spending. We find that only when the old-age labor supply increases with health spending can we find a unique steady state and that the steady-state output per agent decreases with the Social Security contribution rate. Finally, we show that the welfare effects of WRFP are ambiguous. Welfare gains depend on the share of capital in production for the two-period OLG model whereas welfare gains depend not only on the share of capital in production but also on the old-age labor supply for the three-period OLG model.

Acknowledgments

This research is supported by the national social science foundation of China(Grant No.13BGL113).

Appendix

A.1. Proof of Proposition 1

Ignoring the time subscripts and making use of the capital market clearing condition (12), it follows that the dynamics at the steady state can be expressed as

$$(1+\lambda)k = s = \frac{\beta(1-\tau)}{1+\beta}w - \frac{\lambda+\tau}{(1+\beta)}\frac{1-\alpha}{\alpha}k$$

Given this equation, an easy calculation yields:

$$[\alpha(1+\beta)+\tau(1-\alpha)+(1+\alpha\beta)\lambda]k = A\alpha\beta(1-\tau)k^{\alpha} \quad \text{(A.1)}$$

Since the two curves, $y=[\alpha(1+\beta)+\tau(1-\alpha)+(1+\alpha\beta)\lambda]k$ and $y = A\alpha\beta(1-\tau)k^{\alpha}$, have a unique positive point of intersection, it is easy to show that there exists a unique positive steady state solution k^* for (A.1).

Rewrite (A.1) at the steady state k^* as

$$k^* = \left(\frac{A\alpha\beta(1-\tau)}{(1+\alpha\beta)(\lambda+\Omega)}\right)^{1/(1-\alpha)} \quad \text{(A.2)}$$

where,.

$$\Omega = \frac{\alpha(1+\beta)+\tau(1-\alpha)}{1+\alpha\beta} \quad \text{(A.3)}$$

It is evident from Equation (A.2) that $\frac{dk^*}{d\lambda} < 0$ holds.

A.2. Proof of Proposition 2

Note that (A.2) can be rewritten as

$$k^{\alpha-1} = \frac{B_2}{B_1} + \frac{B_3}{B_1}\lambda\left(B_4 k^{\alpha}\right) \quad \text{(A.4)}$$

Define $J(k) = k^{\alpha-1} - \frac{B_2}{B_1} - \frac{B_3}{B_1}\lambda\left(B_4 k^{\alpha}\right)$.Since $J(k)$ is a continuous function and

$\lim_{k\to 0} J(k) = +\infty$, $\lim_{k\to\infty} J(k) = -\frac{B_2}{B_1} - \frac{B_3}{B_1}\tilde{\lambda} < 0$

there exists a stable and nontrivial stationary equilibrium k^*.At the stable state level of per capita capital k^*, rewrite (A.4) as

$$B_1 k^{\alpha-1} = B_2 + B_3\lambda\left(B_4 k^{\alpha}\right) \quad \text{(A.5)}$$

Differentiating Equation (A.5) with respect to τ and noting that B_1 and B_2 are also functions of τ and $\lambda'(h) > 0$, we have

$$\frac{dk^*}{d\tau} = -\frac{\alpha\beta Ak + (1-\alpha)k^{2-\alpha}}{\alpha B_3 B_4 k\lambda'(h) + B_1(1-\alpha)} < 0$$

Hence we obtain Proposition 2.

A.3. Proof of Proposition 3

Differentiating $\bar{U}(\lambda)$ with respect to λ gives

$$\frac{d\bar{U}(\lambda)}{d\lambda} = \frac{1+\beta}{c}\frac{dc}{d\lambda} + \frac{\beta}{1+r}\frac{dr}{d\lambda}$$

By substituting the expressions of the first order conditions into the above equation, the following expression is obtained:

$$\frac{d\bar{U}(\lambda)}{d\lambda} = (1+\beta)\frac{\alpha(1-\tau)\frac{dw}{d\lambda} + (1-\alpha)k + (1-\alpha)(\tau+\lambda)\frac{dk}{d\lambda}}{\alpha(1-\tau)w + (1-\alpha)(\tau+\lambda)k} + \frac{\beta}{1+r}\frac{(1-\alpha)(1+r)}{1+\lambda}$$

Substituting (20) and (22) into (A.6) gives

$$\frac{d\bar{U}(\lambda)}{d\lambda} = (1+\beta)\frac{-\alpha^2(1-\tau)\frac{w}{1+\lambda} + (1-\alpha)k - (1-\alpha)(\tau+\lambda)\frac{k}{1+\lambda}}{\alpha(1-\tau)w + (1-\alpha)(\tau+\lambda)k} + \frac{\beta(1-\alpha)}{1+\lambda}$$

$$= \frac{1+\beta}{1+\lambda}\frac{-\alpha^2(1-\tau)\frac{w}{k} + (1-\alpha)(1-\tau)}{\alpha(1-\tau)\frac{w}{k} + (1-\alpha)(\tau+\lambda)} + \frac{\beta(1-\alpha)}{1+\lambda} \qquad \text{(A.7)}$$

After algebraic manipulation, Equation (A.7) becomes

$$\frac{d\bar{U}(\lambda)}{d\lambda} = \frac{1+\beta}{1+\lambda}\frac{-\alpha^2(1-\tau)\frac{1-\alpha}{\alpha}(1+r)+(1-\alpha)(1-\tau)}{\alpha(1-\tau)\frac{1-\alpha}{\alpha}(1+r)+(1-\alpha)(\tau+\lambda)} + \frac{\beta(1-\alpha)}{1+\lambda}$$

$$\frac{1+\beta}{1+\lambda}\left(\frac{-\alpha(1-\tau)(1+r)+(1-\tau)}{(1-\tau)(1+r)+(\tau+\lambda)}+\frac{\beta}{1+\beta}(1-\alpha)\right)$$

Let

$$g(\alpha)=\frac{-\alpha(1-\tau)(1+r)+(1-\tau)}{(1-\tau)(1+r)+(\tau+\lambda)}+\frac{\beta}{1+\beta}(1-\alpha) \qquad \text{(A.8)}$$

Then we have

$$\frac{d\bar{U}(\lambda)}{d\lambda} = \frac{1+\beta}{1+\lambda}g(\alpha)$$

Since $g(\alpha)$ is a linear decreasing function of α and

$$g(0)=\frac{1-\tau}{(1-\tau)(1+r)+(\tau+\lambda)}+\frac{\beta}{1+\beta}>0$$

$$g(1)=\frac{-r(1-\tau)}{(1-\tau)(1+r)+(\tau+\lambda)}<0$$

it follows that there exists a unique α^* satisfying $g(\alpha^*)=0$ and
$g(\alpha)<0$ if $\alpha>\alpha^*$ and $g(\alpha)>0$ if $\alpha<\alpha^*$
The proposition follows from the above result.

A.4. Proof of Proposition 4

Rewrite (43) as

$$\frac{d\widetilde{U}(\lambda)}{d\lambda}=\frac{\beta}{1+\lambda}\times\left(\frac{(1+\lambda)r+\lambda(1-\alpha)(1+r)}{1+\lambda r}\right.$$

$$\left.-(1-\mu)\frac{(1+\lambda)r+(1-\lambda)(1-\alpha)(1+r)}{1+(1-\lambda)r}+\frac{\alpha(1+1/\beta)(1-\tau)(1-\alpha)(1+r)}{(1+\lambda)\alpha-(1-\tau)(1-\alpha)(1+r)}\right) \quad \text{(A.9)}$$

Define the second term of the left hand side of Equation (A.9) as $f(\mu)$, thus we should only verify $f(\mu)>0$ in order to prove Proposition 4. It is straightforward to show that

$$f(\mu)=\frac{(1+\lambda)r+(1-\lambda)(1-\alpha)(1+r)}{1+(1-\lambda)r}\mu+\frac{(1+\lambda)r+\lambda(1-\alpha)(1+r)}{1+\lambda r}$$

$$-\frac{(1+\lambda)r+(1-\lambda)(1-\alpha)(1+r)}{1+(1-\lambda)r}+\frac{\alpha(1+1/\beta)(1-\tau)(1-\alpha)(1+r)}{(1+\lambda)\alpha-(1-\tau)(1-\alpha)(1+r)}$$

$$=\frac{(1+\lambda)r+(1-\lambda)(1-\alpha)(1+r)}{1+(1-\lambda)r}\mu+l(\tau)$$

Since $f(\mu)$ is an increasing linear function on $[0,1]$, we only should verify $l(\tau)>0$, where

$$l(\tau)=\frac{(1+\lambda)r^2+(2\lambda-1)(1-\alpha)(1+r)}{(1+\lambda r)\big(1+(1-\lambda)r\big)}+\frac{\alpha(1+1/\beta)(1-\alpha)(1+r)}{(1+\lambda)\alpha\frac{1}{1-\tau}-(1-\alpha)(1+r)}$$

Since $\lambda>\frac{1}{2}$, the first term of $g(\tau)$ is larger than 0. Let $\frac{1}{1-\tau}=\varphi$ and

$$h(\varphi)=(1+\lambda)\alpha\varphi-(1-\alpha)(1+r)$$

Since $\varphi\in[1,+\infty)$ and $h(\varphi)$ is increasing on $[1,+\infty)$, we should only verify

$$h(1)=(1+\lambda)\alpha-(1-\alpha)(1+r)>0$$

This inequality holds as an immediate consequence of the inequalities $\lambda > \frac{1}{2} > r$ and $\alpha > \frac{1}{2}$. Thus the second term of $l(\tau)$ is also larger than 0. Then Proposition 4 follows.

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